

Constraints on the interacting holographic dark energy model

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Abstract

We examined the interacting holographic dark energy model in a universe with spatial curvature. Using the near-flatness condition and requiring that the universe is experiencing an accelerated expansion, we have constrained the parameter space of the model and found that the model can accommodate a transition of the dark energy from $\omega_D > -1$ to $\omega_D < -1$.

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Numerous observational results indicate that our universe is undergoing an accelerated expansion driven by a yet unknown dark energy (DE) [1]. While the leading interpretation of such a DE is a cosmological constant whose equation of state (EOS) is $\frac{p}{\rho} \equiv \omega_D = -1$, other conjectures relate DE to a so-called quintessence, with $\omega_D > -1$, or yet to an exotic field with $\omega_D < -1$ [2]. An extensive analysis finds that the current data favors DE models with EOS in the vicinity of $\omega_D = -1$ [3], straddling the cosmological constant boundary. Further works on the data analysis can be traced in [4]. Recently, the analysis of the type Ia supernova data indicates that the time varying DE gives a better fit than a cosmological constant [5], which mildly favor the evolution of the DE EOS from $w_D > -1$ to $w_D < -1$ at a recent stage. Theoretical attempts towards understanding of the ω_D crossing -1 phenomenon have taken place [6].

In a recent work we proposed a holographic DE model [22] with interaction with matter fields to explain the above transition of the DE [12]. Given the unknown nature of both DE and dark matter (DM), which are two major contents of the universe, one might argue that an entirely independent behavior of DE is very special [13]. Studies on the interaction between DE and DM have been carried out [13, 14, 15]. It was argued that the interaction will influence the perturbation dynamics and could be observable through the lowest multipoles of CMB spectrum [14]. Investigation of the interaction between DE and DM in the holographic DE model has been done by using the Hubble scale as IR cutoff to explain the acceleration of our universe [16]. In [12], we extended the inclusion of interaction between DE and DM into the holographic DE model with the future event horizon as an IR cutoff. As a result, we found that our model, with the interaction between DE and DM, can give an early deceleration and late a time acceleration. In addition, the appropriate coupling between DE and DM accommodates the transition of the DE equation of state from $w_D > -1$ to $w_D < -1$. This property could serve as an observable feature of the interaction between DE and DM, in addition to its influence on the small l CMB spectrum argued in [13].

In this paper we would like to extend our previous discussion [12] to a universe with spatial curvature (see also [17]). The tendency of preferring a closed universe appeared in a suite of CMB experiments [18]. The improved precision from WMAP provides further confidence, showing that a closed universe with positively curved space is marginally preferred [19]. In addition to CMB, recently the spatial geometry of the universe was probed by supernova measurements of the cubic correction to the luminosity distance [20], where a closed universe is also marginally favored. At present, the ratio of the sum of the densities of all forms of matter energy in the universe to the critical density required for spatial flatness is $\Omega_{T,0} = 1.02 \pm 0.02$. We will use this “near flatness” property together with the transition of the EOS of DE to constrain our model parameters.

The total energy density is $\rho = \rho_m + \rho_D$, where ρ_m is the energy density of matter and ρ_D is the energy density of the DE. The total energy density satisfies a conservation law. However since we consider the interaction between DE and DM, ρ_m, ρ_D do not conserve separately. They must rather enter the energy balances [16][12]

$$\dot{\rho}_m + 3H\rho_m = Q \quad , \quad (1)$$

$$\dot{\rho}_D + 3H(1 + w_D)\rho_D = -Q \quad , \quad (2)$$

where w_D is the equation of state of DE, Q denotes the interaction term and can be taken as $Q = 3b^2 H \rho$ with b^2 the coupling constant [18]. This expression for the interaction term was first introduced in the study of the suitable coupling between a quintessence scalar field and a pressureless cold dark matter field [13]. The choice of the interaction between both components was meant to get a scaling solution to the coincidence problem such that the universe approaches a stationary stage in which the ratio of dark energy and dark matter becomes a constant. In the context of holographic DE model, this form of interaction was derived from the choice of Hubble scale as the IR cutoff [16].

From (1), (2) and the Friedmann equation with positive curvature $\Omega_m + \Omega_D = 1 + \Omega_k$, where $\Omega_m = \rho_m/(3H^2)$, $\Omega_D = \rho_D/(3H^2)$ and $\Omega_k = 1/(aH)^2$, we get the equation of state of DE,

$$\omega_D = \frac{\Omega'_k}{3(1 + \Omega_k - \Omega_D)} - \frac{(1 + \Omega_k)\Omega'_D}{3\Omega_D(1 + \Omega_k - \Omega_D)} - \frac{b^2(1 + \Omega_k)^2}{\Omega_D(1 + \Omega_k - \Omega_D)} \quad , \quad (3)$$

where the prime denotes the derivative with respect to $x = \ln a$.

The holographic DE in the closed universe is expressed as $\rho_D = 3c^2 L^{-2}$ where c^2 is a constant [8] and with the event horizon as the IR cutoff. Here $L = ap(t)$, where $p(t)$ is defined by $\int_0^{p(t)} \frac{dp}{\sqrt{1-kp^2}} = \int_t^\infty \frac{dt}{a} = R_h/a$ or $p(t) = \sin y/\sqrt{k}$, where $y = \sqrt{k}R_h/a$. R_h is the radial size of the event horizon and L is the radius of the event horizon measured on the sphere of the horizon.

The DE density can also be written as $\rho_D = 3c^2 L^{-2} = \Omega_D 3H^2$. Thus we have $L = \frac{c}{H\sqrt{\Omega_D}} = a \sin y/\sqrt{k}$. Taking the derivative with respect to t on both sides of such an equation, we have

$$-\cos y = -\frac{c}{\sqrt{\Omega_D}}(1 + \dot{H}/H^2) - \frac{c\dot{\Omega}_D}{2\Omega_D^{3/2}H} \quad (4)$$

Deriving now the Friedmann equation with respect to t and using eqs(1,2), we have

$$\dot{H}/H^2 = \frac{-3\Omega_D(1 + r + \omega_D)/2 - \dot{\Omega}_k/(2H)}{1 + \Omega_k} \quad , \quad (5)$$

where r is the ratio of the energy densities, $r = \rho_m/\rho_D = (1 + \Omega_k - \Omega_D)/\Omega_D$. Substituting (5) into (4) and considering the expression of ω_D we got before, we obtain the evolution behavior of the

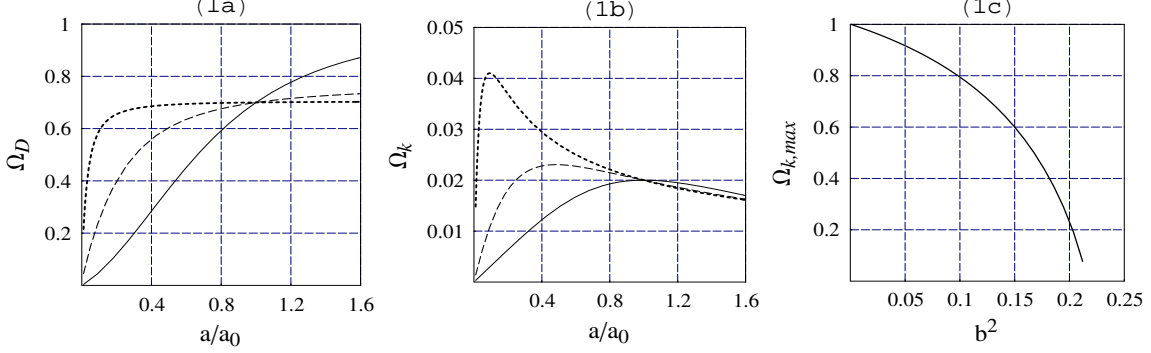


Figure 1: In part *a* we see the evolution of Ω_D in terms of the change of the coupling between DE and DM for a small fixed value of c . In *b* the evolution of Ω_k is shown in terms of the change of b^2 for fixed small c . In *c* the locations of peaks of Ω_k with the change of b^2 for the fixed small c are pictured.

dark energy,

$$\frac{\Omega'_D}{\Omega_D^2} = \frac{1 + \Omega_k - \Omega_D}{1 + \Omega_k} \left[\frac{2 \cos y}{c \sqrt{\Omega_D}} + \frac{1}{\Omega_D} + \frac{\Omega'_k}{\Omega_D(1 + \Omega_k - \Omega_D)} - \frac{3b^2(1 + \Omega_k)}{\Omega_D(1 + \Omega_k - \Omega_D)} \right] . \quad (6)$$

Neglecting the interaction between DE and DM, namely $b^2 = 0$, this result leads to (31) in [8] if we substitute $\Omega_k = aq\Omega_m$ where $q = \Omega_{k0}/\Omega_{m0}$. If we keep b^2 but neglect the curvature of the universe, this expression returns to (5) of reference [12].

With the expression of Ω'_D/Ω_D^2 , we can rewrite (3) in the form

$$\omega_D = -1/3 - 2\sqrt{\Omega_D} \cos y / (3c) - b^2(1 + \Omega_k)(1 - \Omega_D) / [\Omega_D(1 + \Omega_k - \Omega_D)] , \quad (7)$$

where $\cos y = \sqrt{1 - c^2 \Omega_k / \Omega_D}$. In the derivation of (7), we have employed

$$\Omega'_k = -2\Omega_k - 2\Omega_k \times (H'/H) \quad (8)$$

and

$$H'/H = -\frac{3\Omega_D(1 + r + \omega_D)}{2(1 + \Omega_k)} - \frac{\Omega'_k}{2(1 + \Omega_k)} = -3\Omega_D(1 + r + \omega_D)/2 + \Omega_k . \quad (9)$$

With these equations at hand, we are in a position to study the evolution behaviors of different forms of matter energy in the universe. We have two parameters in the evolution equations, namely b^2 indicating the coupling between the DE/DM and c^2 coming from the holography. Different values of b^2 and c^2 influence a lot the evolution behavior of our universe.

Considering the Gibbons-Hawking entropy in a closed universe, $S = \pi L^2$, we require $c^2 \geq \Omega_D/(1 + \Omega_k)$ to satisfy the second law of thermodynamics [8]. By choosing the present values, such that $\Omega_{D0} = 0.7, \Omega_{K0} = 0.02$, we need $c \geq 0.83$. For fixed small values of c (but $c \geq 0.83$),

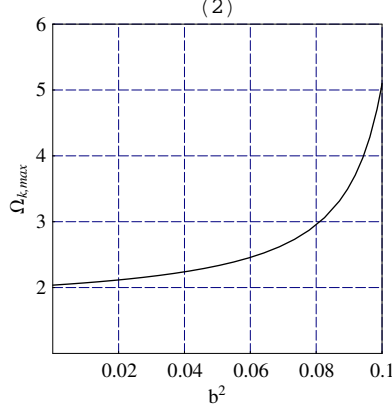


Figure 2: Locations of peaks with the change of b^2 for a fixed big value of c .

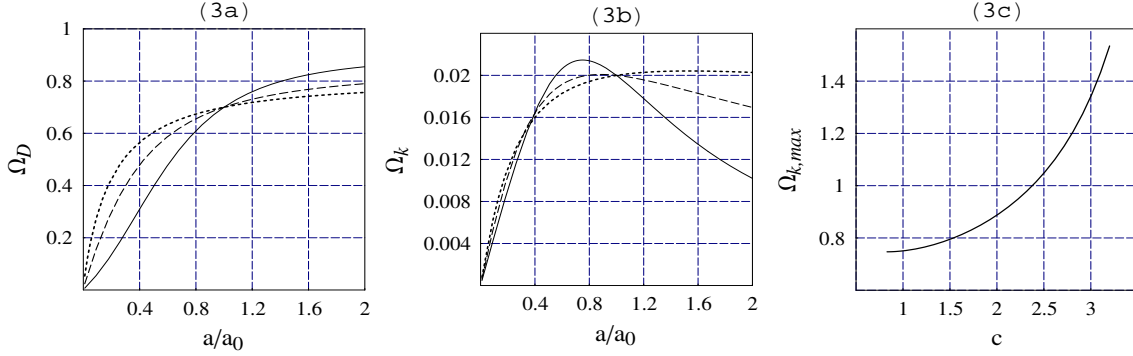


Figure 3: In the part *a* we show the evolution of Ω_D with the change of c for fixed small coupling between DE and DM. In *b* we show the evolution of Ω_k with the change of c for fixed small value of b^2 . Part *c* exhibits the locations of peaks with the change of c .

we observe that with the increase of b^2 , the peaks of Ω_k appear at a smaller scale of the universe and peaks increase with the increase of b^2 . Since $\Omega_k = 1/(aH)^2 = 1/\dot{a}^2$, the location of $\Omega_{k,max}$ corresponds to the minimum value of \dot{a} , which is the starting point of the acceleration ($\ddot{a} = 0$). To describe the present accelerated expansion of our universe, the location of the peak of Ω_k should appear before the present scale, $a \leq a_0$. This gives the lower bound on the value of the coupling between DE/DM.

On the other hand, by taking account of the Friedmann equation $(\dot{a}/a)^2 = (1+r)\rho_D/3 - 1/a^2$, we have $\Omega_T = 1 + 1/\dot{a}^2 = [1 - 1/f(a)]^{-1}$, where we defined the function $f(a) = (1+r)H^2\Omega_D a^2 = 1 + 1/\Omega_k$. Noting that Ω_T is close to one since the early universe, when $a \rightarrow 0$, which indicates that the second term in square brackets in Ω_T must be small, we may expand the expression $\Omega_T \doteq 1 + \Omega_k/(1 + \Omega_k) = (\Omega_k + \Omega_T)/\Omega_T$. This corresponds to requiring that $\Omega_k \ll 1$ at any time.

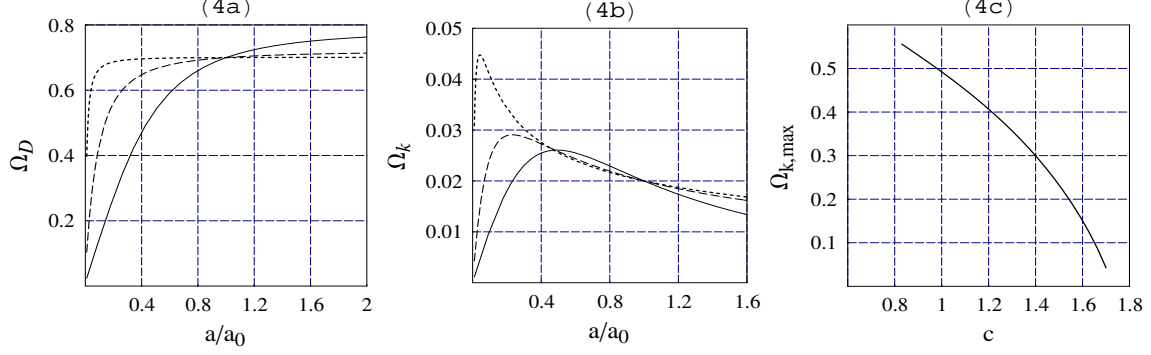


Figure 4: Figure 4a shows the evolution behavior of Ω_D with the change of c for a large value of b^2 . Figure 4b, shows the behavior of Ω_k with the change of c , and figure 4c exhibits the locations of peaks of Ω_k change with the change of c .

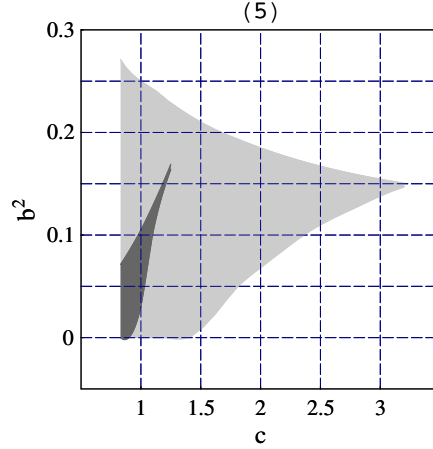


Figure 5: This figure shows the constrained parameter space of b^2 and c .

Using this “near-flatness” condition and putting by hand that $\Omega_{k,max} \leq 0.04$, we can obtain the allowed b^2_{max} which satisfies the “near-flatness” condition for fixed small values of c .

The characters discussed above are shown in Fig.1. Fig.1a is the evolution of the DE. Fig.1b shows the behavior of Ω_k . We see that with larger coupling b^2 , the DE dominates earlier, so that the acceleration starts earlier. Fig.1c shows the location of the maximum value of Ω_k with the change of b^2 . It is clear that when $b^2 = 0$, $\Omega_{k,max}$ appears at $a/a_0 = 1.08$ for $c = 1.5$, which shows that to enter the accelerated expansion before the present time, the interaction between DE and DM is required. The minimum coupling between DE and DM to drive the universe in the accelerated expansion is $b^2 = 0.05$ when $c = 1.5$.

With the increase of the value of c , we observed that allowed range of b^2 by conditions we mentioned above becomes small. For $c > 3.2$, we find that for all values of b^2 , $\Omega_{k,max}$ appears after

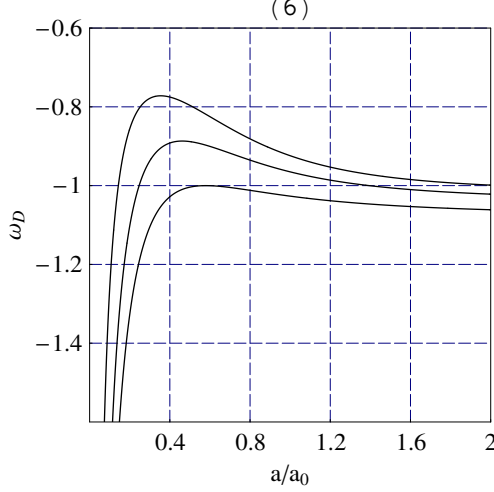


Figure 6: Here we show the behavior of ω_D which crosses the value -1.

the present scale a_0 , which is shown in Fig.2. Therefore, in order to accommodate the acceleration starting before the present time, we found the maximum value of c as given by $c_{max} = 3.2$.

Numerically we have also observed the evolution behaviors for fixed small coupling between DE and DM with the change of the constant c , see Fig.3. We found that with the increase of c , the position of the peaks of Ω_k move to appear at a larger scale a . Requiring that the acceleration begins before the present time, we get the allowed c_{max} for the fixed small b^2 , say $c_{max} = 2.4$ for $b^2 = 0.1$. Combining the lower bound of c from the second law of thermodynamics, for fixed small value of b^2 , we have the parameter space of the constant c .

The property described above changes drastically when $b^2 > b_{cr}^2 = 0.14$. For fixed $b^2 > 0.14$, the results are exhibited in Fig.4. We saw that with the increase of c , the peaks of Ω_k appear for smaller values of a and values of $\Omega_{k,max}$ increases with the increase of c . The permitted c_{max} can be gotten by using the “near-flatness” condition, while c_{min} can still be gotten from the second law of thermodynamics. With the increase of b^2 , the allowed parameter space of c becomes smaller. The range of c vanishes when b^2 reaches $b_{max}^2 = 0.31$.

The allowed parameters’ space of b^2 and c discussed above satisfying the “near-flatness” condition and accommodating the accelerated expansion of our universe happened before the present era is shown in the yellow area of Fig.5.

In order to explain the recent observation that DE experiences a transition from $\omega_D > -1$ to $\omega_D < -1$, we have further constrained the parameters’ space of b^2 and c . For given c , we found that the DE transition can happen earlier for stronger coupling b^2 between DE and DM, which can be seen from Fig.6. If c is bigger, the allowed b_{min}^2 that accommodates this DE transition

increases. On the other hand, for $c > 1.2$, ω_D will only tend to -1 from above but never crosses it when $a \rightarrow \infty$ for any values of b^2 . The parameter space is shown in the red region in Fig.5. If the future more accurate observations can tell us the exact location of this DE transition happened, this parameter space can be further constrained. The value of c around one ($c \in [0.83, 1.2)$) is interesting, since this could serve as a support that the IR regulator might be simply related to the future event horizon. With the future more precise data, this question can be answered exactly.

We also have to point out that the interaction between DE and DM may change some properties of DM clumping. However, this can happen only at a larger time scale, namely after they "thermalize". The question is whether there has been time for both to thermalize or not. We think that they have not! Indeed, the two sectors interact weakly, with an interaction that contains Hubble constant therefore the scale of thermalization should be very large. Today there has certainly not have passed enough time for that. In fact, we know that the expansion is quite recent, thus we are actually at the beginning of the DE dominated era. The interaction cannot change the smooth property of the DE, which is, to our mind, an observational fact however it will influence the clumpy behavior of the DM, certainly not yet thermalized. After the transition from $w_D > -1$ to $w_D < -1$, DM might presumably become smoother than before. This question however has to be focused in the framework of structure formation.

With the interaction between the DE and DM, neither of them can evolve separately. The interaction alters the evolution of matter perturbation and the formation of cosmological structure. The study of the evolution of sub-Hubble linear perturbations in the universe with the DE coupled to DM has been carried out [21] and it was found that the perturbation grows for $w_D > -1$, while it is always suppressed in the $w_D < -1$ case. We expect that this result will also hold in our model.

In summary, we have extended our interacting holographic DE model [13] to the universe with spatial curvature. By imposing the "near-flatness" condition and requiring that at the present era we are experiencing the accelerated expansion, we have obtained the parameter space of the coupling between DE and DM and the constant c from holography. To accommodate the transition of DE from $\omega_D > -1$ to $\omega_D < -1$, we have further constrained the parameter space on b^2 and c . Furthermore we have obtained the results in a closed universe, which is a case mildly favored by recent analysis [10, 19].

Acknowledgments

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